

Now, the ZT of $x_{int}[n]$ can be obtained as

$$\begin{aligned} X_{int}(z) &= \sum_{n=-\infty}^{\infty} x_{int}[n] z^{-n} \\ &= \sum_{n=-\infty}^{\infty} c[n] x[n] z^{-n} \quad \text{--- (4)} \end{aligned}$$

By using (3),

$$\begin{aligned} X_{int}(z) &= \sum_{n=-\infty}^{\infty} \left\{ \frac{1}{M} \sum_{k=0}^{M-1} e^{+jk \frac{2\pi}{M} n} \right\} x[n] z^{-n} \\ &= \frac{1}{M} \sum_{k=0}^{M-1} \left\{ \sum_{n=-\infty}^{\infty} x[n] z^{-n} e^{+jk \frac{2\pi}{M} n} \right\} \\ &= \frac{1}{M} \sum_{k=0}^{M-1} \left\{ \sum_{n=-\infty}^{\infty} x[n] \left(e^{-jk \frac{2\pi}{M}} z \right)^{-n} \right\} \\ &= \frac{1}{M} \sum_{k=0}^{M-1} X \left(e^{-jk \frac{2\pi}{M}} z \right) \quad \text{--- (5)} \end{aligned}$$

Therefore, the ZT of $y[n]$ can be expressed by using (1) and (5) as

$$\begin{aligned} Y(z) &= X_{int}(z^{1/M}) \\ &= \frac{1}{M} \sum_{k=0}^{M-1} X \left(e^{-jk \frac{2\pi}{M}} z^{1/M} \right) \end{aligned}$$

$$\begin{aligned} \Rightarrow Y(e^{j\Omega}) &= \frac{1}{M} \sum_{k=0}^{M-1} X \left(e^{-jk \frac{2\pi}{M}} e^{j \frac{\Omega}{M}} \right) \\ &= \frac{1}{M} \sum_{k=0}^{M-1} X \left(e^{j \left(\frac{\Omega - 2\pi k}{M} \right)} \right) \end{aligned}$$

$$\& \boxed{Y(\Omega) = \frac{1}{M} \sum_{k=0}^{M-1} X \left(\frac{\Omega - 2\pi k}{M} \right)}$$

$$\begin{aligned} \text{For } M=2, \quad Y(\Omega) &= \frac{1}{2} \sum_{k=0}^1 X \left(\frac{\Omega - 2\pi k}{2} \right) = \frac{1}{2} \left[X \left(\frac{\Omega}{2} \right) + X \left(\frac{\Omega - 2\pi}{2} \right) \right] \\ &= \frac{1}{2} \left[X \left(\frac{\Omega}{2} \right) + X \left(\frac{\Omega}{2} - \pi \right) \right] \end{aligned}$$